Effect of Hollow Core on the Rigidity of Fine Fibers

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Synopsis

The principles of torsional and transverse vibrations have been utilized to measure the torsional and bending rigidities of fibers, respectively. A simple device, based on the principle of the torsional pendulum, has been designed to test regular and hollow staple fibers of fine denier. A vibroscope (interfaced with an oscilloscope) was used to measure the linear density of the fibers, and the results were used to calculate their torsional and bending rigidity properties. The effect of hollow core on the torsional and bending rigidities of the fibers is reported.

INTRODUCTION

During the spinning process for spun yarn manufacture and texturing of filaments by false-twisting, the individual fibers and filaments are subjected to bending and twisting. The effectiveness of twisting such a bundle of fibers and the quality of yarn produced from them is governed by the bending and torsional rigidities of the fibers. The objectives of this study were to design a simple device which would measure the rigidity of fibers and to evaluate the effect of hollow core in fibers on their rigidity property. This is achieved by using the principles of torsional and transverse vibrations of a beam. Both hollow and solid fibers were examined and compared for their rigidities.

EARLIER STUDIES

In the past, a number of researchers have worked on the measurements of torsional and bending rigidities of fibers and yarn.[1-10] Several methods have been utilized to measure these properties: the torsional-pendulum method[1-5] and the torque-balance method[6-8] for torsional rigidity; and the cantilever bending method[6,9] and the deformed ring-loop method[10] for bending rigidity.

Pierce[1] was the pioneer in using the torsional-pendulum method to measure the torsional rigidity of cotton fibers. He attached one end of cotton fiber to the center of a small aluminum rod of about 20 mm in length and 1 mm in diameter to form torsional pendulum. Turner and Venkataraman[2] and Meredith[3] used the Pierce's procedure to test the torsional rigidity of cotton and other synthetic fibers. Paul and Bhattacharjee[4] used a torsional pendulum to measure the torsional rigidity of jute fiber bundle. According to them, the bundle test takes care of the variation between

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individual fibers and gives the true average representation of all the fibers. Aghili-Kermani et al.[5] developed an instrument based on torsional pendulum principle to detect damping levels in free oscillation. Chapman[6] used an electrobalance to measure the amount of torque developed in a twisted fiber under tension in order to calculate its torsional rigidity. Dhingra and Postle[7] and Okabayashi et al.[8] developed a device to obtain a torque-twist curve in order to calculate the torsional rigidity of fibers and yarn.

Chapman[6] obtained the bending stress–strain curve by deforming a fine fiber of gage length 1–2 mm. He used an electrobalance to determine the bending force required to bend the fiber, and the bending stress–strain curve to determine the bending rigidity of fibers. Sen[9] devised an instrument to place 150 fibers parallel to one another in 20 mm width and then, he measured the load required to bend these fibers to calculate their flexural rigidity. Hunter et al.[10] employed the deformed ring-loop method for testing the flexural rigidity of a yarn. They obtained an empirical relation by running regression analysis of the test data from single and two-ply yarns.

THEORY

Torsional Rigidity

If a fiber of length l and torsional rigidity GJ is attached at one end to a disc having mass moment of inertia I_m and subjected to a torsional vibration, the equation of motion of free oscillation of fiber can be written as in Ref. 11:

$$I_m\ddot{\theta} + (\mathrm{GJ}/l)\,\theta = 0 \tag{1}$$

where θ is the angular displacement and $\ddot{\theta}$ is the angular acceleration of disc at any instant. The above equation is the differential equation for simple oscillation of the disc, and its solution yields

$$\omega_n = \sqrt{\mathrm{GJ}/I_m l} \qquad \mathrm{rad/s} \tag{2}$$

where ω_n is the angular natural frequency of oscillation. If f is the natural frequency of oscillation and t is the time period per oscillation, then

$$\omega_n = 2\pi f = 2\pi/t \tag{3}$$

Equating eq. (2) and (3) and simplifying gives

$$\mathrm{GJ} = 4\pi^2 I_m l/t^2 \tag{4}$$

If I_m is in kg · mm², l is in mm, and t is in seconds, then GJ will be in mN · mm². The mass moment of inertia I_m of the circular disc was calculated from the mass m and radius R of the disc according to the equation

$$I_m = mR^2/2 \tag{5}$$

If m is in kg and R is in mm, then I_m will be in kg · mm².

Bending Rigidity

If a fiber of length l, bending rigidity EI, under axial tension T is subjected to a transverse vibration, the equation of motion of the fiber can be written as in Ref. 12:

$$\rho_m \frac{\partial^2 y}{\partial t^2} = T\frac{\partial^2 y}{\partial x^2} - \mathrm{EI}\ \frac{\partial^4 y}{\partial x^4} \tag{6}$$

where x is the dimension along the fiber axis and y is the displacement in lateral direction. The solution of the above equation for a simply supported fiber between the two supported points will yield

$$\omega_n = \frac{\pi}{l}\left(\frac{T}{\rho_m}\right)^{½}\left(1 + \frac{\pi^2}{l^2}\cdot\frac{\mathrm{EI}}{T}\right)^{½} \tag{7}$$

If f is the natural frequency of vibration, then

$$f = \frac{1}{2l}\left(\frac{T}{\rho_m}\right)^{½}\left(1 + \frac{\pi^2}{l^2}\cdot\frac{\mathrm{EI}}{T}\right)^{½} \tag{8}$$

Simplifying the above equation for EI gives

$$\mathrm{EI} = \frac{4\ \rho_m f^2 l^4}{\pi^2} - \frac{Tl^2}{\pi^2} \tag{9}$$

If ρ_m is in kg/mm, T is in mN, l is in mm, and f is in cycles/s, then EI will be in mN · mm^2. If the fiber has a tex of n_t, then the linear density ρ_m can be calculated as

$$\rho_m = n_t \times 10^{-9} \tag{10}$$

MATERIALS AND METHODS

Materials Used

The fibers used for testing were of 0.61–1.67 tex and 51–76 mm in length. These fibers were made of polyester, and supplied by Dupont and Eastman. Low-tex hollow fibers were of 0.61 tex and solid fibers of 0.67 and 0.72 tex. High-tex hollow and solid fibers were of 1.67 tex. These fibers are generally used as fiberfill for stuffing in textile products, such as pillows, cushions, sleeping bags, etc.

The average stress–strain curves of these fibers are shown in Figures 1–4, and their properties are given in Table I. Twenty samples of each of these fibers were tested on Instron for their load-elongation behavior. All fibers were tested with a 2.5-cm gage length and were stretched at a rate of 2.5 cm/min. The figures and the table show the average of 20 test results. The specific stress–strain curves of low and high tex Kodel fibers are shown in Figures 1 and 2, respectively, and of low and high tex dacron fibers in Figures 3 and 4, respectively.

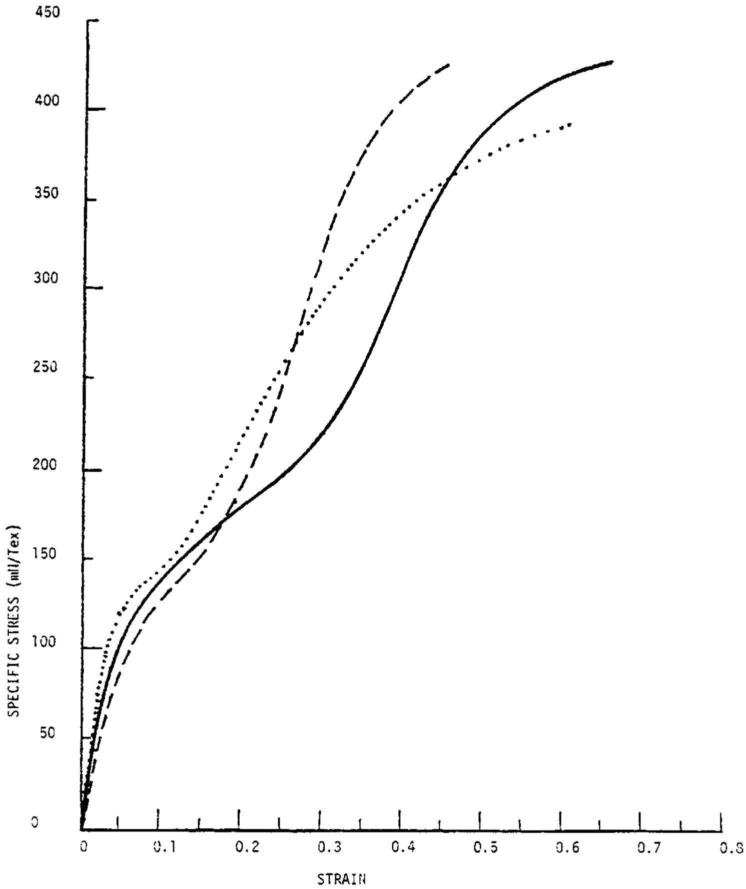

Fig. 1. Stress–strain curves of low tex, regular and hollow, Kodel polyester staple fibers: (——)Kodel 431, regular, 0.72 tex, 51 mm; (---)Kodel 435, hollow, 0.61 tex, 51 mm; (···)Kodel 436, hollow, 0.61 tex, 51 mm.

Torsional Rigidity

Apparatus. The device used to test fibers for their torsional rigidity property is shown in Figure 5. This device was designed to test fine staple fibers and provide atmosphere free of outside air drafts, thus avoiding any disturbance to fiber during testing. The walls and top of the box were made of 5-mm-thick transparent acrylic plastic so the fiber could be observed during testing. Dimensions of the box were chosen to test fibers of up to 25-cm staple length. A mirror with scale was mounted on the back wall to measure fiber length. A bolt, with a hook at its end, was mounted in top of the box to support the fiber sample (which was glued to a tab for mounting). A flat plate was attached to the hook to avoid any possible rotation of the tab, and sample, at the top.

Method. The testing arrangement of fiber sample is shown in Figure 6. A flat circular disc of 3.5 g weight and 14.3 mm radius was fixed at its center to one end of the fiber, and a plastic tab, with hole at the top for hanging the specimen, was glued to the other end of the fiber with a cellulose

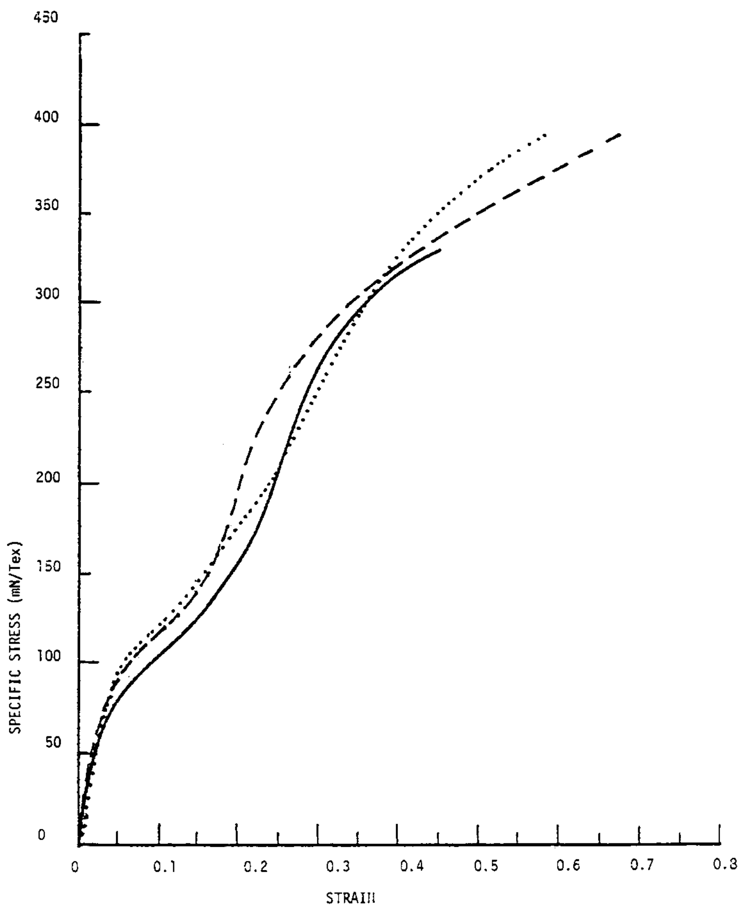

Fig. 2. Stress–strain curves of 1.67 tex, regular and hollow, Kodel polyester staple fibers: (——)Kodel 431, regular, 1.67 tex, 76 mm; (---)Kodel 432, regular, 1.67 tex, 76 mm; (···)Kodel 437, hollow, 1.67 tex, 76 mm.

nitrate glue solution. A very fine hole (little larger than the fiber diameter) was drilled in the center of the disc to avoid any off balance oscillation of disc during torsional vibration. The motion of the tab during testing was restricted by the flat plate attached to the hook. The following steps were followed to test the fiber samples:

- With the help of tweezers, a sample of single fiber was taken at random from the fiber bundle.
- One end of the fiber was poked through the central hole of the disc and glued on the other side. (A radial line had been previously drawn on the disc to aid in following its rotation.)
- The other end of the fiber was glued to a plastic tab which had a hole at its top for hanging the specimen.
- After allowing the glue to dry, the disc was checked for balance about the fiber axis.
- The sample was hung in the testing chamber and allowed to come to rest.

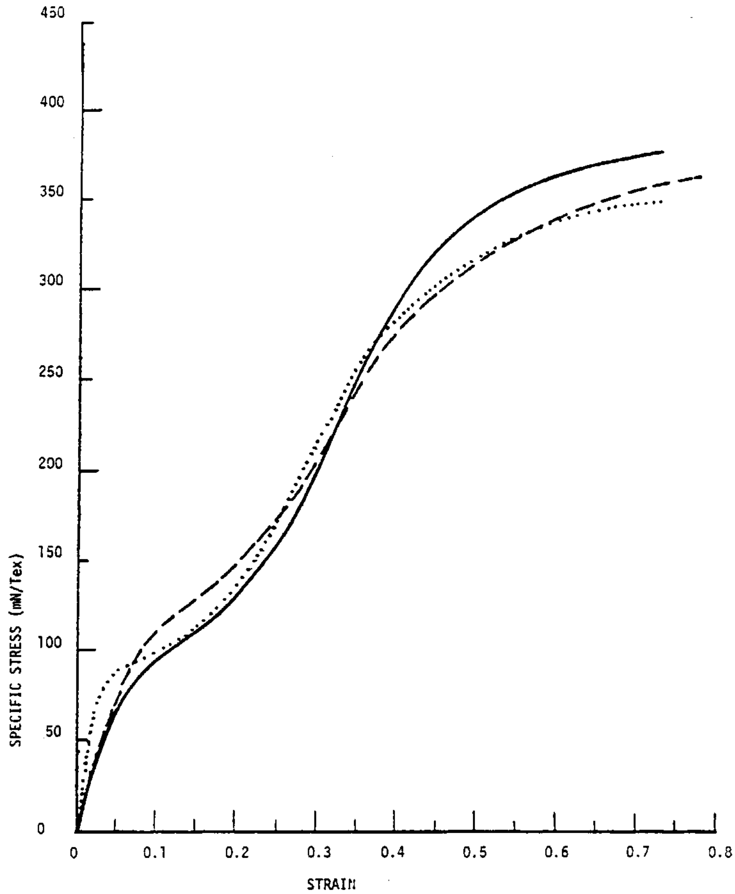

Fig. 3. Stress–strain curves of low tex, regular and hollow, Dacron polyester staple fibers: (——) Dacron UBP, regular, 0.67 tex, 51 mm; (---) Dacron 808, hollow, 0.61 tex, 51 mm; (···) Dacron II, hollow, 0.61 tex, 51 mm.

- With the help of the bolt at the top, the sample was given a turn of small magnitude.
- After a couple of initial torsional vibrations, the time required for five complete cycles of oscillation was noted.
- The time period for one oscillation was calculated.
- After aligning the line of sight, fiber sample and its image in the mirror, the length of the fiber sample was measured from the scale mounted on the mirror.
- Torsional rigidity of the fiber sample was calculated by using eq. (4).

Bending Rigidity

Apparatus. The fiber samples were tested on vibroscope, by using standard procedure, to measure their linear density in denier. The vibroscope gives transverse vibration to a sample of fiber. A Tektronic Oscilloscope #468 was attached to the vibroscope to read its natural frequency of transverse vibration. The vibroscope was set up for 19.761 mm gage length of

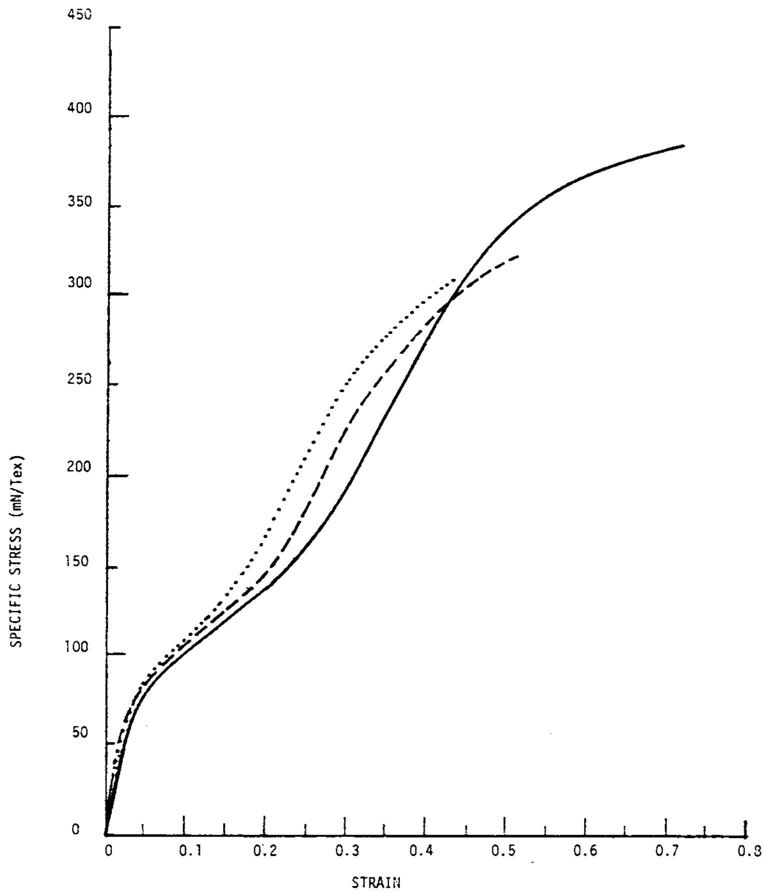

Fig. 4. Stress–strain curves of 1.67 tex, regular and hollow, Dacron polyester staple fibers: (——) Dacron UBP, regular, 1.67 tex, 76 mm; (---) Dacron 76, hollow, 1.67 tex, 76 mm; (···) Dacron 91, hollow, 1.67 tex, 76 mm.

fibers and a tension of 7.636 mN was applied to the low-tex fibers and 18.956 mN to the high-tex fibers during testing.

Method. The portion of the fiber, which was subjected to transverse vibration, was held between two notches. This may be considered as a case of simply supported beam as shown in Figure 7. To prepare a fiber sample for testing on vibroscope, plastic tabs were glued to both ends of the fiber by Duco-Cement. One end of the sample was hung on the vibroscope and the tension was applied to the other end. The sample was subjected to the natural frequency of vibration by gradually increasing the rate of transverse vibration. The denier value of the fiber was read from the dial on the vibroscope, and the natural frequency of vibration from the oscilloscope. Twenty fibers were selected at random from each set of the fiber samples and tested for their natural frequency of transverse vibration.

The linear density of the fibers was calculated from their tex values by using eq. (10). Knowing the gage length, tension, linear density, and the natural frequency of transverse vibration of the fibers, the bending rigidity of the fibers was obtained by using eq. (9).

TABLE I
Tensile Properties of Regular and Hollow Staple Polyester Fibers

Fiber type	Hollow/ regular	Tex (specified)	Staple length (mm)	Tenacity (mN/tex)	Breaking elongation (%)	Initial modulus (mN/tex)
Kodel 431	Regular	0.72	51	430	65.0	2305
Kodel 435	Hollow	0.61	51	430	45.0	2102
Kodel 436	Hollow	0.61	51	397	59.8	2773
Kodel 431	Regular	1.67	76	331	44.6	2128
Kodel 432	Regular	1.67	76	403	66.7	2146
Kodel 437	Hollow	1.67	76	396	57.6	2243
Dacron UBP	Regular	0.67	51	379	70.5	1536
Dacron 808	Hollow	0.61	51	367	76.9	1669
Dacron II	Hollow	0.61	51	353	72.5	1845
Dacron UBP	Regular	1.67	76	392	71.4	2402
Dacron 76	Hollow	1.67	76	327	51.2	2331
Dacron 91	Hollow	1.67	76	315	44.4	2402

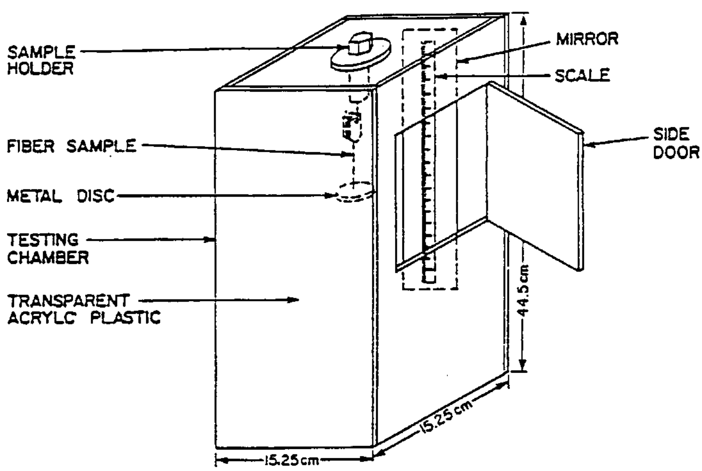

Fig. 5. Apparatus for measuring torsional rigidity.

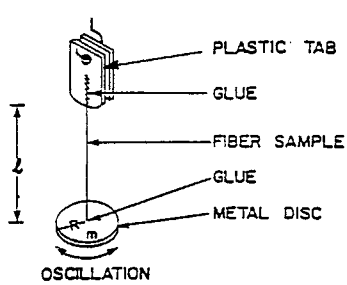

Fig. 6. Method of testing torsional rigidity.

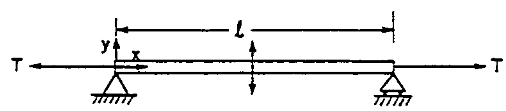

Fig. 7. Method of testing bending rigidity.

RESULTS AND DISCUSSIONS

The torsional and bending rigidities of the fibers and their corresponding measured tex values are given in Table II. Each value represents the average of 20 tests. Photomicrographs of the fiber cross sections revealed that the opening in the center of a hollow fiber was about 15% of the total cross section.

Comparison of torsional and bending rigidities of low-tex with high-tex polyester fibers shows an increase in rigidity with increase in fiber tex. This is because rigidity increases with the fourth power of fiber diameter, and therefore with the square of fiber tex.

The general trend of the data is that hollow fibers show a greater rigidity than solid fibers of similar linear density. This is expected because when fibers have the same linear density, the hollow fiber has a larger outside diameter, even though their area of solid cross section is the same. The larger diameter resulting from a hollow core will produce a larger moment-of-inertia. The increase in rigidity of hollow fibers, therefore, depends upon the size of hollow core.

The deviation from expected behavior for Kodel 435 and 436 could be due to lower linear density of hollow fibers than compared to the solid ones. Also, the bending rigidity of a fiber was calculated by using the measured tex value of the fiber rather than the value specified by the supplier. It was observed that eq. (10), which is used to calculate the bending rigidity of fiber, is quite sensitive to the value of linear density of the fiber.

TABLE II
Rigidity Properties of Regular and Hollow Staple Polyester Fibers

Fiber type	Regular/ hollow	Tex (measured)	Torsional rigidity (mN mm^2)	Bending rigidity (mN mm^2)
Kodel 431	Regular	0.70854	0.038	0.065
Kodel 435	Hollow	0.61009	0.042	0.059
Kodel 436	Hollow	0.65086	0.032	0.083
Kodel 431	Regular	1.39698	0.156	0.304
Kodel 432	Regular	1.67420	0.177	0.365
Kodel 437	Hollow	1.45170	0.213	0.353
Dacron UBP	Regular	0.68073	0.034	0.041
Dacron 808	Hollow	0.56451	0.035	0.043
Dacron II	Hollow	0.57541	0.037	0.048
Dacron UBP	Regular	1.58933	0.199	0.382
Dacron 76	Hollow	1.56099	0.236	0.394
Dacron 91	Hollow	1.59980	0.214	0.415

If the other values in the equation are kept constant, then a 1% deviation in linear density will change the bending rigidity by 50%. Thus, random variation among fibers and small measurement errors could cause significant changes in calculated rigidity values. There are also factors other than tex and cross section (i.e., surface finish, morphology, etc.) which affect rigidity, but these have not been considered here.

CONCLUSION

The device designed for testing the torsional rigidity and the method developed to test the bending rigidity of fine staple fibers provides easy and economical method of testing the fibers. It is very important to know the accurate tex value of the fiber to calculate its bending rigidity by the equation developed in this work. As expected, hollow fibers of same tex as regular fibers are found to have higher rigidity than those of regular fibers.

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